STATISTICAL ANALYSIS OF REFEREE ASSIGNMENTS AND BOUT OUTCOMES SENIOR SABER EVENTS 2022-2024 STEPHEN G. BRONARS AND CAROL MA EDGEWORTH ECONOMICS MAY 17, 2024

Background and Qualifications

Stephen Bronars is a Partner with Edgeworth Economics, a consulting firm specializing in economic and statistics research. He worked previously at Welch Consulting for more than eight years and prior to that was the Leroy Denman Jr. Regents Professor of Economics at the University of Texas at Austin. He has submitted expert reports and testified on damages and liability in discrimination cases, analyses of class certification and collective action topics, and on statistical analyses and sampling in other labor and employment matters. He works extensively on a consulting basis to analyze data relating to labor and employment issues. He earned a PhD in Economics from the University of Chicago. He has published peer-reviewed articles on labor economics, econometrics, and applied statistics in academic journals and has written articles for trade publications on topics related to the statistical analysis of human resource data.

Carol Ma is a Managing Consultant at Edgeworth Economics. She specializes in conducting and managing statistical analyses of labor and employment issues and other data analytics projects. She received her Master's Degree in Asset Management from Yale School of Management and her BA in Economics and Neuroscience, *summa cum laude*, from the University of Chicago.

Assignment

Edgeworth Economics was hired by USA Fencing to conduct a statistical analysis of referee assignments and competition outcomes in senior (Division I) saber competitions for the period in which selections were made for positions on the 2024 U.S. Olympic Saber Team. One purpose of the statistical analysis is to identify statistical anomalies in referee assignments for both U.S. events and FIE overseas events. A second purpose of the analysis is to test whether there are significant differences in the outcomes of American fencers related to the referees who are officiating their bouts in the pool rounds of FIE overseas events. Our statistical analysis focuses on pool rounds in FIE events when points were awarded for selection to the Paris Olympic Team because (i) there may have been less scrutiny of bouts in pool rounds, (ii) unlike later direct elimination rounds where more experienced and qualified referees receive assignments, the assignment of referees to pools should mirror a random assignment process that limits conflicts of interest, and (iii) an excellent performance by an American fencer in a pool round can help them to be exempt from the preliminary direct elimination round or receive better direct elimination placement and generate valuable points towards the selection for the Olympic Team.

Our assignment was to:

- Identify whether referee assignments in senior saber events resulted in a significantly higher frequency of encounters between certain fencers and referees relative to an assignment pattern consistent with a random selection and assignment process that attempts to minimize possible conflicts of interest.
- Identify whether senior American fencers had significantly more successful performances in pool rounds of FIE saber events when officiated by certain

referees as compared to their performances when officiated by other referees in other FIE events.

Summary of Conclusions

The analyses described in this report focus on referee assignments and competition outcomes in the pool rounds of FIE events in the Paris Olympic Team selection year and the prior year. The outcome of a pool round is important for determining a fencer's seeding in the direct elimination rounds and the ability to generate points that could contribute to selection for the Olympic Team. Irregularities in either referee assignments or the scoring of bouts in pool rounds could substantially impact the points earned by American fencers competing in FIE events and influence their possible selection for the Olympic Team.¹

Our primary conclusions are:

- There was not a significant and systematic difference in the referee assignment process in pool rounds in the Paris selection year when compared to assignments for the same events in the prior year for both USA National and FIE events, for male and female fencers.
 - The concentration of referee assignments to fencers did not change significantly in the Paris selection year relative to the prior year; fencers were not systematically and significantly more likely to encounter the same referees in multiple events when compared to the prior year.
- Among American fencers only faced referee assignments in pool rounds of FIE events during the Paris selection year that were significantly more

¹ The greater number of referee assignments and bout outcomes in the pool rounds provide more statistical power for our analyses. Future analyses of the data obtained for this study could extend to the larger tables in direct elimination rounds.

concentrated (at the 5% level) than what would have been expected from a random referee assignment process that minimized conflicts of interest.

- and and faced referee assignments in pool rounds of FIE events during the Paris selection year that were significantly more concentrated (at the 10% level) than what would have been expected from a random referee assignment process that minimized conflicts of interest.
- Three American fencers (and and and a second se

performed significantly better in FIE pools in the Paris selection period than fencers to whom they were compared with similar seedings entering FIE events in this period.

- Significantly better performance in FIE pools in the Paris selection period could be due to many factors other than biased treatment by referees.
- Neither Tatiana Nazlymov nor Mitchell Saron performed significantly differently in FIE pools than fencers to whom they were compared with similar seedings entering FIE events in the Paris selection period.
- Among American fencers only Tatiana Nazlymov (when referred by and and and and a statistically significantly (at the 10% level) better in their pool rounds of FIE events in the Paris selection year relative to their performances in other pools of FIE events officiated by other referees in the same year.
 - Mitchell Saron performed nearly significantly better in his pool rounds of FIE events refereed by the selection of the Paris selection year relative

to his performance in other pools of FIE events officiated by other referees in the same year.

- There are no American fencers who had both a significantly higher concentration of referee assignments and performed significantly better in pools officiated by referees who they encountered multiple times in FIE events during the Paris selection year.
 - We find no statistical evidence based on referee assignments or performance outcomes in pool rounds of FIE events that would warrant a reduction in points earned, or elimination of points earned, for FIE events during the Paris selection year for any American fencer.
- Because Mitchell Saron earned over 2,000 points more than the fourth ranked fencer on the USA Senior Team Point Standings as of April 28th, 2024 for men's saber, even if the points that he earned in FIE events in which he was refereed by
 in the pool round were reduced or completely eliminated, he would remain the third ranked male fencer in saber.
- Although Tatiana Nazlymov performed significantly better in pools of FIE events officiated by **Example 1** during the Paris selection year relative to her other performances, the concentration of her referee assignments was not significantly different from what was expected so there is insufficient statistical evidence to warrant a reduction in points awarded in those FIE events.
 - Because Tatiana Nazlymov earned only 155 more points towards selection for the Paris Olympic Team than the fourth ranked fencer on the USA Senior Team Point Standings as of April 28th, 2024 for women's saber, any

meaningful reduction in points awarded in her FIE events would likely drop her to the fourth place.

Data Used to Analyze Referee Assignments and Bout Outcomes

Our analyses focus on senior saber individual USA National events and official FIE events in the 12 months in which points were accumulated to qualify for the 2024 Olympic Team and during the previous 12 months for the same competitions.² USA National-level Division I events are based on the National Tournament Results published on the USA Fencing website³ and senior saber individual FIE events are identified in previous competition results on the FIE website.⁴

The match-level data, collected from publicly available online results operated by three

major fencing tournament management systems (Fencing Time,⁵ Engarde,⁶ and Ophardt Team

Sportevent⁷), encompass 20 USA National events and 36 FIE Grand Prix, World Championship,

² Selection criteria for 2024 Senior Men's and Women's Saber Olympic Games are published on the USA Fencing website (<u>https://www.usafencing.org/selection-criteria</u>). The top three athletes on the National Senior Team Point Standings as of April 28, 2024 were selected as members of the 2024 Olympic Team and the fourth athlete on the National Senior Team Point Standings was selected as the replacement athlete. Points stay on the point standings until the competition is held again, or if it is not held again, those points are dropped one year after it was held.

The National Senior Team Point Standings for women's saber as of April 28, 2024 count points from 9 FIE Grand Prix, World Cup, and World Championship events held between April 28, 2023 (Grand Prix Seoul 2023) and March 15, 2024 (World Cup Sint-Niklaas 2024). The National Senior Team Point Standings for men's saber as of April 28, 2024 count points from 9 FIE Grand Prix, World Cup, and World Championship events held between April 27, 2023 (Grand Prix Seoul 2023) and March 22, 2024 (World Cup Budapest 2024).

The USA domestic events that count towards the National Senior Team Point Standings for men's and women's saber as of April 28, 2024 are Division I NACs held in October 2023, December 2023, January 2024, and April 2024. For completion of USA National Division I events analyzed in the rolling 12-month period and consistency with events counting towards the National Senior Point Standings (NRPS), we also included the Summer Nationals in July 2023 in USA National events for the Paris selection period.

³ <u>https://www.usafencing.org/natresults</u>

⁴ <u>https://fie.org/competitions</u>

⁵ <u>https://www.fencingtimelive.com/</u>

⁶ <u>https://engarde-service.com/</u>

⁷ <u>https://www.fencingworldwide.com/en/</u>

and World Cup events held in the period from May 2022 to April 2024.⁸ The list of all events analyzed with the data source links and tournament information is included in **Appendix A**.

Data collected for each event includes (1) event format (i.e., number of pools, number of direct elimination rounds, and number of fencers promoted to the next round), (2) the list of fencers competing in the event with their pre-tournament ranking and affiliation information, (3) fencers' seeding for the pool round, (4) pool scores for each bout and referees, strip, and time scheduled for each pool, (5) results from the pool round (i.e., victory / match, touches hit, and touches received) and rankings after the pool round, (6) scores for each direct elimination bout and referee for each bout, and (7) final ranking of the fencers in the event.

In addition to fencer and referee information provided on Fencing Time Live, Engarde, and Ophardt Team we gathered current referee ratings and division information from USA Fencing⁹ and FIE¹⁰ websites for referees in analyzed events.

In total for pool rounds, we collected data from 1,315 pools for 8,746 competitor counts¹¹ in 56 USA and FIE events. These include five USA events in the Paris selection period and five more in the prior year, and nine FIE events in the Paris selection period and nine more in the prior year, for both men and women. **Exhibit 1** provides some descriptive statistics on pool rounds analyzed. For example, **Exhibit 1** shows that for USA National events for women, matches included bouts in 15 to 24 pools in the Paris selection period with a median of 17 pools, and 16 to 24 pools in the 12 months before the Paris selection period, with 20 pools in the median match. **Exhibit 1** also shows that for USA National events for women during the Paris

⁸ A fencing season runs from August 1st to July 31st. *See* Footnote 2 for rationale of events included. ⁹ https://member.usafencing.org/referees

¹⁰ https://fie.org/referees

¹¹ 10 pools (pool #1 - #10) for 70 fencers in the Tunis Grand Prix Senior Men's Saber event held on January 13-15, 2023 do not contain referee information. Based on video evidence provided by Saron, we manually assigned as the referee to pool #3 in the 2023 Tunis Grand Prix Senior Men's Saber event in our analysis.

selection period, matches included 102 to 161 fencers and 13 to 24 referees in the pool rounds.

The median USA National match for women had 118 fencers and 21 referees in the pool round.

EXHIBIT 1:

					Women	's Events					Men's	Events		
Event			Events		25th		75th		Events		25th		75th	
Category	Period	Statistic	Analyzed	Min	Percentile	Median	Percentile	Max	Analyzed	Min	Percentile	Median	Percentile	Max
[a]	[b]	[c]	[d]	[e]	[f]	[g]	[h]	[i]	[j]	[k]	[1]	[m]	[n]	[0]
		Number of Pools	5	16	18	20	21	24	5	22	22	25	29	31
	12 Months	Number of Fencers in the Pool Round	5	107	124	128	144	161	5	144	146	167	199	212
	before the Paris	Number of Referees in the Pool Round	5	15	23	23	25	26	5	18	20	24	24	26
	Selection Period	Average Number of Referees Per Pool	5	1.67	1.81	1.81	1.85	1.88	5	1.00	1.28	1.29	1.95	1.95
UCA Notional		Number of Scheduled Time Slots	5	2	2	2	2	2	5	2	2	2	2	2
USA Ivational		Number of Pools	5	15	17	17	22	24	5	20	23	27	31	32
	Duming the Domis	Number of Fencers in the Pool Round	5	102	118	118	149	161	5	139	150	178	212	219
	During the Paris	Number of Referees in the Pool Round	5	13	17	21	22	24	5	18	18	24	25	26
	Selection Period	Average Number of Referees Per Pool	5	1.53	1.55	1.83	2.00	2.00	5	1.30	1.52	1.63	1.65	1.96
		Number of Scheduled Time Slots	5	2	2	2	2	2	5	2	2	2	2	2
		Number of Pools	9	17	20	21	23	25	9	19	23	27	28	31
	12 Months	Number of Fencers in the Pool Round	9	106	120	133	161	167	9	121	127	184	189	210
	before the Paris	Number of Referees in the Pool Round	9	13	13	18	18	23	8	13	13.5	17.5	19	23
FIE Grand Prix	Selection Period	Average Number of Referees Per Pool	9	1.00	1.12	1.35	2.00	2.35	8	1.00	1.11	1.46	1.91	2.04
and World		Number of Scheduled Time Slots	9	1	2	2	3	3	9	1	2	3	3	3
Championship/		Number of Pools	9	17	19	20	24	27	9	21	23	26	32	36
Cup	During the Durin	Number of Fencers in the Pool Round	9	86	133	139	160	167	9	120	160	181	219	250
	During the Paris	Number of Referees in the Pool Round	9	12	13	14	21	22	9	13	15	17	22	23
	Selection Period	Average Number of Referees Per Pool	9	1.00	1.18	1.40	1.89	2.00	9	1.00	1.31	1.46	1.54	1.93
		Number of Scheduled Time Slots	9	1	2	2	3	3	9	1	2	3	3	4

SUMMARY OF THE POOL ROUNDS IN EVENTS ANALYZED

Exhibit 2 shows the distribution of the number of events each fencer attended among USA National events and FIE events analyzed and the average seeding positions of the fencers. During the Paris selection period 28.1% (64 of 228) of female fencers and 27.1% (88 of 325) of male fencers in USA National events participated in only one event. Fencers who participated in one USA event are, on average, seeded at about the 75th percentile for women and the 73rd percentile for men. 37.2% (128 of 344) of female fencers and 36.7% (179 of 488) of male fencers in the FIE Grand Prix, World Championship, and World Cup events we analyzed participated in only one event during the Paris selection period. Fencers who participated in one FIE event are, on average, seeded at about the 89th and 85th percentile for women and men respectively. Statistical analysis of the referee assignments and performance outcomes for fencers who participated in very few events is not feasible. **Exhibit 2** indicates that the fencers who will be excluded from our statistical analysis are seeded much lower than the fencers for

whom we analyze both referee assignments and bout outcomes. In both USA National events and FIE events, higher ranked fencers, as reflected by their average seeding percentile, tend to participate in more events.

EXHIBIT 2:

DISTRIBUTION OF THE NUMBER OF EVENTS EACH FENCER ATTENDED

		12 Ma	onth Before the	Paris Selection	Period	During the Paris Selection Period					
		Women	s Events Men's Events		Women	s Events	Men's Events				
	Number of		Average		Average		Average		Average		
	Pool Rounds	Number of	Seeding	Number of	Seeding	Number of	Seeding	Number of	Seeding		
Event Category	Attended	Fencers	Percentile	Fencers	Percentile	Fencers	Percentile	Fencers	Percentile		
[a]	[b]	[c]	[d]	[e]	[f]	[g]	[h]	[i]	[j]		
	1	61	71.6%	86	66.3%	64	74.6%	88	73.3%		
LICA Notional	2-3	79	62.5%	99	58.1%	76	61.9%	123	58.1%		
USA National	4-5	89	42.5%	118	45.0%	88	42.0%	114	43.3%		
	Total	229	50.4%	303	50.3%	228	50.4%	325	50.3%		
	1	115	83.6%	172	84.3%	128	88.9%	179	84.8%		
FIE Grand Prix	2-3	91	68.2%	116	67.0%	75	67.6%	134	73.1%		
and World	4-5	44	61.8%	64	58.9%	40	57.6%	48	59.3%		
Championship/C	6-7	51	44.4%	55	44.3%	43	50.9%	56	40.6%		
up	8-9	44	30.5%	44	21.4%	58	31.9%	71	31.9%		
-	Total	345	50.4%	451	50.2%	344	50.4%	488	50.3%		

Simulations for Analyzing Referee Assignments

A benchmark for the expected outcomes from random neutral referee assignments is needed to determine whether a fencer's multiple encounters with the same referee in pool rounds across different events is significantly different from our expectations. We simulate random referee assignments with restricted conflicts of interest¹² for each male and female fencer in the Paris selection period, and the year before the selection period, for the pool rounds of USA National events and FIE events for which referee assignment data are available. The framework for each simulation matches the actual events and pool times in which fencers competed. For example, suppose a fencer competed in two FIE events that each had eight pools, and eight

¹² Conflicts of interest are expected to be minimized in the referee assignment process. We consider the nationality of the referees recorded on the tournament management platforms for FIE events and the division and club information of the referees for USA National events.

referees officiating those pools at the time the fencer competed. At each event the fencer has a one in eight chance of encountering each referee if assignments were made at random without considering conflicts of interest between the referee and fencers in the pools. In addition, if the same eight referees officiated both events, there would be 64 possible combinations of fencer-referee assignments and only eight would result in the fencer facing the same referee twice.¹³ After conflicts of interest are taken into account, suppose there are only seven referees in the first event and five referees in the second event, for a total of 35 combinations for which the assigned referees in the second event are among the seven available for the first event, there would be five combinations out of 35 possible in which the fencer encounters the same referee twice.

Conflicts between the referee and all the fencers in each pool are avoided if possible and limited otherwise in our simulations. For some assignments avoiding conflicts may be impossible so our process in those cases is to choose assignments with the fewest conflicts. For example, suppose all pools except two at an event and pool time include a fencer from Spain, and there are five Spanish referees who must be assigned to the pools at this time. Our approach is to keep assignments in which the fewest possible Spanish referees are assigned to pools with Spanish fencers.¹⁵

¹³ If some of the referees differed between the two events the likelihood of a fencer encountering the same referee twice would be lower. If there were two referees officiating a pool, our simulations also assigned a second referee to that pool.

¹⁴ When conflicts of interest are accounted for the list of referees that a fencer is likely to encounter depends on the characteristics of other fencers in the same pool. An American fencer competing in a pool that includes several fencers from Spain, for example, is unlikely to encounter a Spanish referee. Our simulations account for possible conflicts from all fencers assigned to each pool.

¹⁵ The 2022 Madrid World Cup for men had 31 pools (held at 3 pool times). There are a total of 13 referees in the pool round and 5 of them (

are Spanish. 24 of the 31 pools have a Spanish fencer. In the actual event, 9 pools with a Spanish fencer were assigned a Spanish referee, 1 pool with an Italian fencer was assigned an Italian referee, and 1 pool with a German fencer was assigned a German referee.

Using the method of randomly assigning referees to pools while limiting conflicts at events and times described above, we simulated the assignment of referees to pools in all the events in **Exhibit 1** for which we observed actual referee assignments. For each fencer we constructed the exact distribution of referee encounters that would occur with random referee assignments with conflict restrictions. The distribution describes the pattern of unique referees that a fencer could face (e.g., one referee encountered three times and four referees encountered once across five events) with the associated likelihood of observing each set of encounters. The simulations are repeated for fencers competing in multiple events with the full referee selection and assignment process repeated 10,000 times for each event category and period (e.g., women's USA National events during the Paris selection period). We simulate the pairing of fencers and referees in pool rounds for all fencers in all events for which we have information about the referees who were present. Based on the simulated frequency of referee-fencer pairings for the events and pool times in which the fencer participated, we have constructed an estimate of the likelihood that the fencer's actual referee pairings, and other possible referee pairings, would have occurred if referee assignments were made through a random selection process with conflict restrictions.

Exhibit 3 provides an example to illustrate a referee assignment pattern in actual data. In the five USA National events we analyzed during the Paris selection period, **Competed in pool** rounds in four events. Two pools in which **Competed were** officiated by one referee and two pools were officiated by a single referee. In total, he encountered five unique referees in his pool rounds. His pool round was officiated by one of the five referees twice (**Competed methods** and the four other referees once. Given the number of unique fencers and referees in our data, and the number of events entered by the typical fencer, a

study of the frequency of unique individual fencer-referee pairings would have low statistical power and would likely generate inconclusive results.¹⁶ Our approach is instead to focus on the concentration of referee assignments for each fencer based on the events he or she attended.

EXHIBIT 3:

EXAMPLE OF REFEREE-FENCER PAIRINGS IN USA NATIONAL EVENTS PARIS SELECTION PERIOD APRIL 2023-APRIL 2024 Fencer Name Referee Name Events Attended in the Category Referee Encounters [a] [b] [c] [d]

We order the possible referee-fencer pairings by the highest number of repeat encounters

that occurred, and then by the second highest number of repeat encounters, and so on.

referee assignments in **Exhibit 3** are summarized as (2,1,1,1,1) because he encountered one referee twice and four referees only once. The least concentrated set of referee assignments possible for would be (1,1,1,1,1,1), in which he would encounter six different referees and never be officiated by the same referee twice. Examples of more concentrated referee assignments, that could occur for given the referees available for his pool

¹⁶ For example, in the actual data for USA National events for women, we observe 1,100 unique referee-fencer pairings in the Paris selection period.

times and the division and club affiliations of available referees and the fencers in his pool, are $(3,1,1,1), (3,2,1), \text{ and } (2,2,1,1).^{17}$

We rank each pattern of referee assignments that occur in our simulations for each fencer in each event type and period (Paris selection or prior year) from the least concentrated to the most concentrated outcomes. For USA events we use a random process that limits the division and club conflicts between referees and fencers to generate the distribution of simulated assignments. For FIE events we use a random process that limits country-based referee-fencer conflicts to generate the distribution of simulated assignments. Because different fencers competed in different events and competed in pools at different times within those events, the possible patterns of referee assignments differ across fencers. To make comparisons of assignment patterns across fencers we calculate the percentile rank of a fencer's actual referee assignments within all simulated assignment outcomes. For example, the percentile rank of the referee assignments of **Exhibit 3** is 87.7%¹⁸, meaning that his actual assignments were more concentrated than 87.7% of the referee assignments in the simulation outcomes we observed.

If a fencer is paired with the same referee in a disproportionate number of pools the result will be a higher concentration of pool bouts for the fencer. We consider a fencer to have a concentration of referee assignments that is significantly greater than what is expected from random assignments if the percentile rank of the observed set of referee assignments is 95% or higher when compared to simulated outcomes (a significance level of 5%).

¹⁷ Among these five sets of referee assignments, we would order them from most concentrated to least concentrated as: (3,2,1), (3,1,1,1), (2,2,1,1), (2,1,1,1,1), and (1,1,1,1,1,1).

¹⁸ In the simulations of the pool rounds for **Mathematical** USA National events, 77.1% of the referee assignments result in the pattern (1,1,1,1,1,1) or six different referees with no repeated encounters. 21.2% of the times he would encounter a same referee twice and the rest four referees once. The percentile rank for the observed pattern (2,1,1,1,1) is calculated by summing the frequency of all less concentrated patterns and the midpoint of the frequency of the observed pattern (77.1% + 21.2%/2 = 87.7%).

Comparison of Actual Referee Assignments and Simulation Results

The first question we analyze is whether referee-fencer assignments differ significantly overall during the Paris selection year as compared to the prior year. We first calculate the percentile rank of each fencer's referee assignments, relative to the distribution of simulated outcomes from random assignments with restricted conflicts, for all fencers who competed in at least three events in each year and event type. For example, percentile rank in the concentration of her referee assignments in FIE events in the Paris selection year is 67.7% in the seven pools in which she competed and her percentile rank in the prior year is 50.9% in the eight pools in which she competed. We conduct the test by comparing the percentile ranks for fencers' referee assignments in the Paris selection year and during the prior year to determine whether assignments were becoming systematically more or less concentrated during the Paris selection year relative to the prior year. For example, if all fencers experienced the same increase in the percentile rank of their referee assignments as between the prior year and the Paris selection year, our test would find a systematic and significant increase in the concentration of referee assignments.

Exhibit 4 presents the results of four tests of a possible systematic change in the concentration of referee assignments: USA events for men, USA events for women, FIE events for men, and FIE events for women. **Exhibit 4** shows, for example, that we conduct the test for the 166 men and 117 women who competed in three or more USA events in the Paris selection year and the 168 men and 128 women who competed in three or more USA events in the prior year. While there are between four and ten fencers in each year and event type with a percentile rank above 95%, there is no statistical evidence that referee assignments overall were becoming significantly more or less concentrated in the Paris selection year when compared to the prior year. Referee assignments became slightly (but insignificantly) less concentrated in USA events

and more concentrated (but insignificantly) in FIE events during the Paris selection year. The pvalues for the four tests presented in **Exhibit 4** range from .238 (women in FIE events) to .948 (men in FIE events) and are all well above the .05 level required for statistical significance.

EXHIBIT 4:

COMPARISON OF AVERAGE RANK PERCENTILES OF REFEREE ASSIGNMENT CONCENTRATION PARIS SELECTION PERIOD AND THE PREVIOUS YEAR

		12 N	Ionths Before th	e Paris Selection	Period		. Between			
				Fencers in 3 of	Fencers in 3 or More Events			Fencers in 3 of	Two Periods	
Event Category	Gender	Fencer Count	Fencers in 2 or Fewer Events	Average Cumulative Percentile	Fencers with Cum. Pctl. > 95	Fencer Count	Fencers in 2 or Fewer Events	Average Cumulative Percentile	Fencers with Cum. Pctl. > 95	T-Test Result: P- value
[a]	[b]	[c]	[d]	[e]	[f]	[g]	[h]	[i]	[j]	[k]
USA National	F M	229 303	101 135	53.2 50.8	4 8	228 325	101 159	51.1 48.4	5 6	0.464 0.314
FIE Grand Prix and World	F	345	167	48.0	8	344	171	50.9	4	0.238
Championship/Cup	М	451	244	50.9	5	488	276	51.0	10	0.948

The results in **Exhibit 4** indicate that there has not been a systematic change in the concentration of referee-fencer assignments for either USA events or FIE events for men or women. If there was a systematic and widespread effort to pair certain fencers with certain referees in the 12 months during which selection for the USA saber Paris Olympic Team occurred, we would expect a significantly higher concentration of referee assignments in the pool round in the Paris selection year than in the prior year, but we did not observe a significant increase. The largest increase in the concentration of assignments occurred for women in FIE events but the increase was insignificantly different than zero.

We next examine the fencers who competed in the pool round of at least three FIE events and experienced a concentration of referee assignments during the Paris selection year that was significantly higher than expected based on simulated assignments. **Exhibit 5** presents the four female and ten male fencers with an excessive concentration of referee assignments in the pool round in FIE events during the Paris selection year. **Exhibit 5** lists the fencer's name, nationality, gender, number of events attended, referee assignments, and percentile rank of the observed assignments relative to the simulated distribution of outcomes.

Ехнівіт 5:

FENCERS WITH HIGHLY CONCENTRATED REFEREE ASSIGNMENTS POOL ROUNDS OF FIE EVENTS DURING THE PARIS SELECTION YEAR





For example, among female fencers has the highest percentile rank of 98.9% for her referee assignments. She competed in the pool round of six FIE events in the Paris selection year and encountered referee the three times, and referee twice. A pattern of referee encounters at least this concentrated is only observed in about 1.1% of our simulations for FIE events for **Exhibit 5** is **16** is **17** is **17** is **17** is **17** is **17** is **18** is **18** if **18** is **18** if **18** is **18** if **18** i

Exhibit 6 shows the referee assignments in the pool round for all American fencers who competed in three or more FIE events during the Paris selection year. The information in Exhibit 6 is the same as in Exhibit 5 and includes all American fencers regardless of the concentration of referee assignments. Among the 12 female fencers listed, has the highest concentration of referee assignment in the pool rounds at 87.8% because in the six events she attended there were two referees () with whom she had two and encounters. Tatiana Nazlymov is the only other female fencer who encountered more than one referee (and twice. The concentration of referee assignments is at a percentile rank of 74.5% which is well below the threshold that would indicate a significant difference from what was expected based on our simulations of referee assignments for her nine events and times.

EXHIBIT 6:

THE CONCENTRATION OF REFEREE ASSIGNMENTS FOR AMERICAN FENCERS

Events Cumulative Attended List of Referees in Pools Fencer Name Gender Encounters Percentile [a] [b] [c] [d] [e] [f] NAZLYMOV TATIANA F 2 9 74.5 2 1 1 1 1 1 i i

POOL ROUNDS OF FIE EVENTS DURING PARIS SELECTION PERIOD







Among the 14 American male fencers listed in Exhibit 6 three (

and **and and the end of the second of the percentile ranks are high enough, given the limited sample size based on events attended, to warrant further scrutiny. For example, our simulations for Mitchell Saron's referee assignments in the pools were more concentrated than his actual assignments only about 10.2% of the time, indicating that his experience of encountering three different referees twice would occur relatively infrequently if assignments were made at random.**

Measuring Pool Outcomes by the Change in a Fencer's Rank During Pool Rounds

To examine whether fencers' pool round outcomes varied depending on the referee(s) who officiated the pool bouts, we constructed an outcome measure based on the change in the fencer's relative position from the pre-match seeding to the pool ranking. The pool ranking is an informative measure of performance in the pool round given that fencers are ranked first by the ratio of victories to bouts fenced, then by the indicator calculated as the difference between total touches scored and total touches received, and finally by total number of touches scored.¹⁹ First, we calculate each fencer's seeding percentile in percentage terms based on their seeding for the pool round and the fencer's pool ranking percentile in percentage terms based on the placement ranking after the pool round.²⁰ Our measure of the pool outcome is the difference between the

¹⁹ o.73 *FIE Organisation Rules*, November 2023. <u>https://static.fie.org/uploads/32/163445-Organisation%20rules%20ang.pdf</u>

²⁰ The seeding and pool results are limited to fencers who competed in the pool round. For senior FIE World Championships, World Cups, and Grand Prix events, the 16 fencers entered who are ranked highest in the most

fencer's relative ranking in the event before and after the pool round. The change in a fencer's ranking during the pool round summarizes the performance of the fencer during pool bouts relative to pre-match expectations. An outstanding performance in the pool round of an FIE event can generate a pool ranking that might elevate the fencer into the top 32 (including the top 16 seeded fencers who did not compete in a pool) and exempt the fencer from the preliminary Direct Elimination rounds.

Pool Outcomes for American Fencers in Prior Year and Paris Selection Year

Using the measure of pool outcomes described above, we conduct a statistical test, known as the "rank sum" test, to examine the pool outcomes in FIE events for each American fencer compared to fencers who have similar pre-match seedings in the Paris selection year and the prior year separately. For each period, we identify the fencers in six or more pool rounds of FIE events and calculate the average seeding percentile across those events. Then we compare the pool outcomes for each fencer to the eight fencers with closest average seeding percentiles²¹ by ranking the pool outcomes in the period among all nine fencers by the improvement in pool ranking relative to pre-match seeding. The test-statistics (p-value) of the rank sum test reflect the probability of observing the actual ranking of pool outcomes for the American fencer relative to the eight comparator fencers had there been no systematic difference in pool outcomes, on average, among all nine fencers with similar FIE rankings before the events²².

Exhibit 7 presents the test results for four American female fencers and six American male fencers who competed in the pool round of six or more FIE events in both the Paris

recent official FIE classification are exempt from the pool round and the preliminary direct elimination table. o.85 *FIE Organisation Rules*, November 2023.

²¹ Conducting the rank sum tests by including 6 fencers with most similar seedings yields identical results for American fencers at the 10% significance level.

²² The seeding in the FIE events is based on the latest official FIE ranking and by drawing lots among the fencers who are not in the ranking. 0.68, 0.108, *FIE Organisation Rules*, November 2023.

selection period and the prior year. For both the Paris selection period and the prior year, we show the pool rounds the fencer participated in, the average seeding percentile among those events, the average pool outcomes which is calculated as the change from the pre-match seeding percentile to pool ranking percentile, and the rank sum test statistics compared to eight similarly seeded fencers (highlighted for p-values of 10% or less). Significantly better or worse pool outcomes compared to fencers with similar FIE rankings could be a result of an improvement or decline in fencer skills or potentially biased treatment by specific referees in some of the pool rounds that appear to differ from the pool outcomes of other fencers with similar seedings.

EXHIBIT 7:

STATISTICAL TESTS OF POOL RESULTS FOR AMERICAN FENCERS IN FIE EVENTS Relative to Similarly Seeded Fencers

			Months Before	e the Paris Selection I	Period	During the Paris Selection Period					
Fencer Name	Gender	Pool Round Counts	Average Seeding Percentile	Average Pctl Change Seeding to Pool Rank	Rank-sum Test P-Value	Pool Round Counts	Average Seeding Percentile	Average Pctl Change Seeding to Pool Rank	Rank-sum Test P-Value		
[a]	[b]	[c]	[d]	[e]	[f]	[g]	[h]	[i]	[j]		
NAZLYMOV TATIANA	F	9	22.5%	2.0%	0.736	9	12.3%	16.4%			
	-					i					
									21		
SARON MITCHELL	Μ	6	39.3%	-0.7%	0.627	9	10.3%	8.2%			

DURING THE PARIS SELECTION PERIOD AND THE PRIOR YEAR

Among the four female fencers in Exhibit 7, Tatiana Nazlymov and

were on average seeded in the top 23 percent and top 37 percent respectively in the year prior to the Paris selection period and both were seeded in the top 20 percent in the Paris selection year among fencers in the pools. Both Ms. Nazlymov and experienced a decline in their pool ranking relative to the seeding, on average, in both periods but the pool outcomes are not significantly different from other fencers with similar pre-match seedings. they attended and experienced improvements in performance in pool rounds in both periods.

pool outcomes are significantly better than similarly seeded fencers in the year prior to the Paris selection period (p-value = 3.4%) and **pool** outcomes are significantly better than similarly seeded fencers in the Paris selection period (p-value = 0.3%).

Among the six male fencers in **Exhibit 7**, **Constrained** and Mitchell Saron both finished the pool rounds on average at a slightly higher rank percentile than their seedings among fencers in the pools during the year prior to the Paris selection and finished on average at a lower rank percentile than their seedings in the Paris selection year. The pool outcomes are not significantly different from similarly seeded fencers for **Constant** and **Mr**. Saron in both periods. **Constant** and **Constant** had significantly better outcomes in pool rounds compared to fencers with similar seedings in the Paris selection period (p-value = 2.1% and p-value = 5.1%). **Constant** is the only fencer in **Exhibit 7** who was exempt from the pool round in three FIE events in the Paris selection period.²³

To summarize, there were three American fencers (

and who performed significantly better in FIE pools in the Paris selection period than fencers who were similarly seeded entering FIE events in this period. Neither Tatiana Nazlymov nor Mitchell Saron performed significantly differently in FIE pools in the Paris selection period than fencers who were similarly seeded entering FIE events in this period.

Statistical Test of the Relationship Between Referee Assignments and Pool Outcomes

After examining the differences in pool outcomes relative to similarly seeded fencers for American fencers in the two periods, we focus on the Paris selection period to determine whether

²³ See Appendix B-1 and Appendix B-2 for more details of top USA fencers exempt from pools in FIE events in the Paris selection year.

any fencers had systematically better or worse outcomes in pools officiated by referees encountered in multiple events, relative to pool outcomes in other FIE events. To determine whether a fencer performed significantly better in a pool round officiated by a referee, compared to pool rounds officiated by other referees, we conduct another rank sum test by referees for each fencer. If a particular referee is associated with significantly better bout outcomes for a fencer than other referees, the pools in which the referee officiated should have systematically higher measures of the fencer's improvement in ranking relative to the fencer's outcomes in other pools.

Exhibit 8 provides summary statistics and results of rank sum tests for the referee-fencer pairings with the most substantial and favorable difference in outcomes when officiated by certain referees, with a p-value of 10% or less. While none of the comparisons in **Exhibit 8** are below the 5% level of statistical significance, given the limited number of events in which a fencer competes, these favorable differences in outcomes when officiated by certain referees are significant at the 10% level and worth examining.

EXHIBIT 8:

STATISTICAL ANALYSIS OF POOL OUTCOMES FOR FENCER – REFEREE PAIRINGS FAVORABLE OUTCOMES WITH TEST STATISTICS BELOW 0.1

								Seeding to	Pool Rank Percen	tile Change
 Fencer Name	Gender	Fencer's Nationality	USA Fencer	Pools Rounds of FIE Events	Referee Name	Referee's Affliation	Referee Encounters	Average with this Referee	Average During Paris Selection	Rank-sum P-Value
[a]	[o]	[c]	[a]	[e]	[1]	lgj	[H]	[1]	10	[K]
18	-		10				-			
	- E			Ē.			Ē			
5		8	3	3 - 2 - 2			: - :	-		3 -3
				Ī.			Ē			
				<u> </u>						

There are three female fencers, including American fencer Tatiana Nazlymov, and nine male fencers, including American fencer **Construction** listed in **Exhibit 8**. Tatiana Nazlymov competed in nine FIE events and encountered referee **Construction** twice. On average, across the nine events Ms. Nazlymov saw her pool percentile ranking increase by 16.4%, meaning that she exited the typical pool round with a less favorable ranking than what was reflected in her preevent seeding. However, in the two events in which **Construction** was a referee in her pool, her ranking decreased by 16.1% on average. The likelihood of observing this pattern of much more favorable pool outcomes when refereed by **Construction** compared to the other pool outcomes with different referees is only 5.6% if the differences of outcomes based on referee assignments is neutral.

also competed in nine FIE events during the Paris selection year and encountered referee twice. On average, across the nine events saw his pool percentile ranking increase by 18.3%, meaning that he exited the typical pool round with a less favorable ranking than what was reflected in his pre-event seeding. However, in the two events in which was a referee in his pool, his ranking decreased by 15.4% on average. The likelihood of observing this pattern of much more favorable pool outcomes when refereed by compared to the other pool outcomes with different referees is only 5.6% if the differences of pool outcomes based on referee assignments is neutral.²⁴

Exhibit 9 presents the same information as in **Exhibit 8**, for all American fencer-referee pairings for fencers who competed in at least three pool rounds in FIE events during the Paris selection period and for the referees who they encountered more than once. Because some

²⁴ The statistical test results for Tatiana Nazlymov and **provide the set of the statistical test results for Tatiana Nazlymov and provide test statistical test results for Tatiana Nazlymov and the set of test statistical test results for Tatiana Nazlymov and provide test statistical test results for Tatiana Nazlymov and test statistical test results for Tatiana Nazlymov and provide test statistical test results for Tatiana Nazlymov and test statistical test results for Tatiana Nazlymov and provide test statistical test results for Tatiana Nazlymov and test statistical test results for Tatiana Nazlymov and provide test statistical test s**

fencers encountered the same referee more than once for multiple referees, these fencers (Tatiana Mitchell Saron, appear in multiple Nazlymov, and rows of Exhibit 9. In addition, the information in Exhibit 9 is not limited to fencer-referee pairings in which the fencer performed better than in pools officiated by other referees. The final column of **Exhibit 9** shows whether the outcomes of the pools officiated by the referee were better or worse than the other pool outcomes for the fencer. For example, while Tatiana Nazlymov had better outcomes in the two pools officiated by than in her other had worse outcomes in the two pools officiated by seven pools. than in her other five pools. On average, across all seven events saw her pool percentile ranking increase by 33.3%, meaning that she exited the typical pool round with a much more favorable ranking than what was reflected in her pre-event seeding. However, in the two events in which was a referee in her pool, her ranking decreased by only 19.4%. The likelihood of observing the less favorable outcomes experienced by in is only 9.5% if the differences of pool outcomes based on pools officiated by referee assignments is neutral.

Overall, **Exhibit 9** presents nine fencer-referee pairings for female fencers with six better and three worse outcomes for the pairings with repeat encounters, and 12 fencer-referee pairings with seven better and five worse outcomes for the pairings with repeat encounters. The test results in **Exhibit 9** show that in addition to the two American fencers, Tatiana Nazlymov and



officiated by with a 11.1% likelihood of observing the more favorable outcomes experienced by Mr. Saron when paired with **Experience**, if differences in outcomes based on referee assignments is neutral.

EXHIBIT 9:

STATISTICAL ANALYSIS OF POOL OUTCOMES FOR FENCER – REFEREE PAIRINGS USA Fencers During the Paris Selection Period

						Seeding to	Pool Rank Percen	tile Change	
Fencer Name	Gender	Pools Rounds of FIE Events	Referee Name	Referee's Affliation	Referee Encounters	Average with this Referee	Average During Paris Selection	Rank-sum P-Value	Pools with this Referee Compared to the Other Events
[a]	[b]	[¢]	[d]	[e]	[f]	[g]	[h]	[i]	[j]
								-	2
1	i	i i							
		- i i							
							_		_
					i.				
									1
		ii							
			20						
		i i							
		2			2				

The Relationship Between Referee Assignments and Pool Outcomes

This section of the report combines the information from the previous sections to explain that no American fencer experienced both a significantly higher concentration of referee assignments in pool rounds of FIE events during the Paris selection period and had a significantly better performance in pool rounds of FIE events when officiated by a referee who was encountered multiple times during these pool rounds. We require evidence of both a significantly higher concentration of referee assignments and significantly better improvement in a fencer's rank when officiated by referees with multiple encounters to raise a red flag. A fencer may have a significantly higher concentration of referee assignments than what would have been expected, resulting in an unusually high number of encounters with the same referee, but perform no differently or even relatively worse in pools officiated by that referee. This pattern of outcomes does not raise a red flag. Other fencers may perform significantly better in pools officiated by a referee with whom they have multiple encounters. If the repeat encounters are not the result of a significantly higher concentration of referee assignments this would not raise a red flag because the multiple encounters with the same referee would be consistent with the assignments that would have occurred with a random process that limited conflicts.

Among American female fencers who competed in three or more FIE events during the Paris selection period as shown in **Exhibit 6**, ______ is the only fencer with a rank percentile for referee assignment concentration above 80%, and not significantly higher than expected at the 10% level. Her pool outcomes with the two referees that she encountered twice during the Paris selection period as shown in **Exhibit 9** are poorer with ______ (p-value = 0.133) compared to pools with other referees and better with ______ (p-value = 0.267) compared to pools with other referees. The pool outcomes for both _______ fencer-referee pairings are not significantly different than her other outcomes at the 10% level.

For male fencers who competed in three or more FIE events during the Paris selection period, _______ and ______ and _____ experienced significantly more concentrated referee assignments with a rank percentile above 90% as shown in **Exhibit 6**. _______ is the only American fencer who encountered the same referee three times (_________ in the six pool rounds he attended. **Exhibit 9** shows that his pool outcomes are slightly poorer but not significantly different (p-value = 0.7) with _______ compared to pools with other referees. ______ encountered ______ and ______ and ______ twice and

did not receive significantly more favorable or less favorable outcomes with either referee (both with p-value = 0.533). **Example 1** similarly had slightly better but not significantly different pool outcomes in the two pools officiated by **Example 2** (p-value = 0.667).

Among the three American male fencers who experienced a nearly significantly higher concentration of referee assignments (rank percentile between 80% and 90%) in the pool rounds of FIE events in the Paris selection period, Mitchell Saron encountered three referees twice and only achieved better pool outcomes compared to the pools with other referees with

(p-value = 0.111), which is not statistically significant at the 10% level. (p-value = 0.111), which is not significantly different pool outcomes with both referees he encountered twice (p-value = 0.143 with **mathematical** and p-value = 0.643 with **mathematical**) and **mathematical** pool performance are almost the same with **mathematical** who he encountered twice as with other referees (p-value = 1). Overall, there is no statistical evidence at the 10% significance level for significantly more favorable pool performances with referees of multiple encounters among American fencers with even a moderately higher than expected concentration of referee assignments in FIE pools.

The only two American fencers in **Exhibit 8** who performed significantly better (at the 10% level) in FIE pools officiated by a referee with whom they had repeat encounters are Tatiana Nazlymov when refereed by ______ and _____ when refereed by ______ the concentration of referee assignments for Ms. Nazlymov and ______ have rank percentiles of 74.5% and 39.3% respectively, meaning that their incidence of repeated encounters with the same referees are well below what would be required for a significant difference. There is no significant difference between their observed concentration of referee assignments and their

expected concentration of referee assignments based on our simulations of random referee assignments for each fencer's events and times with restricted conflicts.

Results for Tatiana Nazlymov and Mitchell Saron

It is our understanding that questions have been raised on social media platforms about the outcomes of matches in which both Tatiana Nazlymov and Mitchell Saron have competed. These questions concern the validity and accuracy of the scoring decisions in certain matches primarily involving referees **and the second s**

Appendix B-1 and **Appendix B-2** list the top 9 American fencers on the USA Senior Team Point Standings as of April 28, 2024 for men and women respectively. These rankings are relied upon in the selection of the Paris Olympic USA Saber Team. Seeding for the events, referees encountered in the pool rounds, ranking from the pool rounds, and final ranking for all designated FIE events that can contribute points to the Group II of the points on the Senior Team Point Standings are presented by the order of points earned for each of the 18 fencers. Top 16 seeds in each of the FIE events are exempt from the pool round and therefore do not have the pool's referee or pool round ranking information. Column [s] indicates whether the points earned from the FIE event are counted towards the points in Group II for the Senior Team Point Standings. Only the four highest points (maximum of two 33-64 results) from events listed in **Appendix B-1** and **Appendix B-2** are counted for the selection process.

Mr. Saron, ranked third, has a total of 5,923 points including 3,888 from FIE events during the Paris selection period. He earned 1,224 points and 804 points in the two FIE events

where was the referee or one of the referees for his pool rounds. He did not encounter was due to favorable treatment below the suggestion that his selection to the Paris Olympic Team was due to favorable treatment by these referees is not supported by the data we analyzed. He earned points from three FIE (all between 33rd to 64th place) that are not counted towards his Group II total because they generated fewer points than his highest four results. Even if his Group II points were reduced or excluded from the events where was a referee in his pool, he would still have earned 2,416.5 points²⁶ in Group II and 4,451.5 points in total.

Ms. Nazlymov is similarly ranked third on the Women's Senior Team Point Standings. Among all nine designated FIE events, she did not encounter **and the senior** or **and the senior** as a referee for any of her pools. Although there is no allegation directly involving given the statistical results for the fencer – referee pairs in **Exhibit 5**, we examined

²⁵ Referees were mentioned in a New York Times article as having exhibited likely favoritism towards Mr. Saron and Ms. Nazlymov.

https://www.nytimes.com/2024/05/09/world/europe/fencing-olympics-turmoil.html

²⁶ 1,200 from Milan World Championships in July 2023, 660 from Seoul Grand Prix in April 2023, 280 from Alger World Cup in November 2023, and 276.5 from Tbilisi World Cup in February 2024.

Group II points been excluded from the events where her pools were officiated by she would have earned 3,014 points²⁷ in Group II or 4,705 points in total. If her point totals were reduced from these events by more than 155 points she would drop to fourth place because

, ranked fourth, has a total of 5,982 points and **second second**, ranked fifth, has a total of 2,842.7 points. However, there is not sufficient statistical evidence to warrant this point reduction; Ms. Nazlymov's referee assignments in FIE pools are not significantly more concentrated than what would be expected given her FIE events and referees available for her pool times. Absent conclusive statistical evidence there would need to be other evidence such as consistently biased calls based on analyses of bout video recordings to evaluate whether there was potentially favorable treatment of specific fencers by certain referees.

²⁷ 1,224 from Sint-Niklaas World Cup in March 2024, 774 from Orléans Grand Prix in December 2023, 756 from Lima World Cup in February 2024, and 260 from Athènes World Cup in March 2024.